$$b = \frac{a}{8}(12 - D) = \tfrac{3}{8}(9.64975) = 3.61865, \qquad b^2 = 13.095 +$$

c is the distance from the foot of perpendicular a to center of ellipse.
y is the distance from A to center of ellipse.

$$c^2 = b^2\frac{(12 - D)^2 + (10 - D_1)^2}{(12 - D)^2} = 21.8552 +$$

$$y = \sqrt{a^2 + C^2} = \sqrt{30.8552} = 5.5547 +$$

$$R = \frac{r}{a}y = \tfrac{1}{4}y = 1.3887 +$$

Volume of pipe is $8\pi Rr = 2.51328 \times 1.3887 \times 0.75 = 26.17644 +$ cubic feet.
Hence remaining capacity of room is 960 cu. ft. $-$ 26.17644 cu. ft. equal to 933.82356 cubic feet.

2722 [September, 1918]. Proposed by FRANK IRWIN, University of California.

The number of terms in the general polynomial of the nth degree in m variables and in that of the mth degree in n variables is the same. It would be interesting to devise schemes which, without assuming this result, should exhibit the terms of these polynomials in one-to-one correspondence with each other.

Solution by C. F. Gummer, Queen's University.

Consider first the polynomial $P_n(x_1, x_2, \cdots, x_m)$ of degree n, with coefficients all equal to 1. The general term is

$$x_1^{p_1}x_2^{p_2}\cdots x_m^{p_m}x_{m+1}^{p_{m+1}}, \qquad (\Sigma p = n),$$

where $x_{m+1} = 1$. Let the term be written at length, and a y $(= 1)$ inserted after each group of like x's except after the one for x_{m+1}, the y appearing even when the corresponding p is zero. The term is completely defined by the positions of the y's, so that the subscripts may be dropped, and the term written $xx \cdots yxx \cdots yxx \cdots$. Thus, in a polynomial of degree 4 in 5 variables, $x_1^2x_2$ will be denoted by $xxyxyyyyx$, the last x representing $x_6 = 1$. The various terms of P_n then correspond to the permutations of n x's and m y's. In the same way the terms of $P_m(x_1, x_2, \cdots x_n)$ of degree m may be made to correspond to the permutations of m x's and n y's. We may now put into one-to-one correspondence the terms of P_n and P_m which differ by interchange of the letters x and y.

Since the choice of a term in P_n corresponds to the choice of positions for the m y's, this method furnishes a direct explanation of the fact that the number of combinations of $m + 1$ kinds of thing taking n things at a time and allowing repetition is $\binom{m+n}{n}$.

2729 [November, 1918]. Proposed by N. P. PANDYA, Sojitra, India.

Solve in integers $x^3 + 3y^4 = z^2$.

Solution by S. A. Corey, Des Moines, Iowa.

Having obtained by any means one solution x, y, z, it is easily seen that a^4x, a^3y, a^6z is a solution, where a may be any integer. Since 1, 2, 7 and 1, 1, 2 are solutions, a^4, $2a^3$, $7a^6$ and a^4, a^3, $2a^6$ are solutions, whatever the value of a.

2731 [November, 1918]. Proposed by J. K. WHITTEMORE, Yale University.

A bowl is in the form of a paraboloid of revolution. If for a given volume the surface is a minimum, prove that the ratio of the diameter of the top to the depth is approximately 1.86.

Solution by Elijah Swift, University of Vermont.

Let the parabola have the equation, $y = kx^2$. Call the depth of the bowl, l. The required ratio is